Box Spline Reconstruction on the Face-Centered Cubic Lattice

Minho Kim, Alireza Entezari and Jörg Peters

IEEE Visualization 2008
23 October

UF UNIVERSITY of FLORIDA

Overview

Overview

reconstruction

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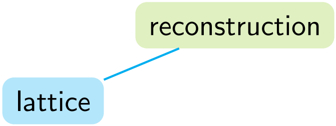

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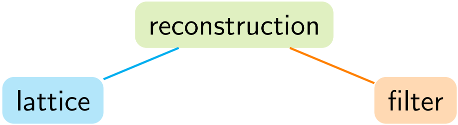

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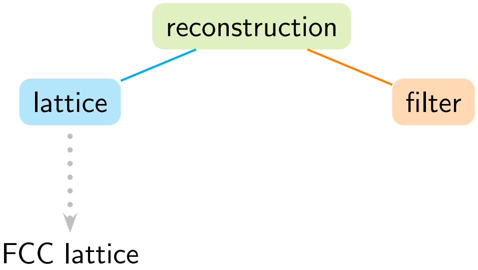

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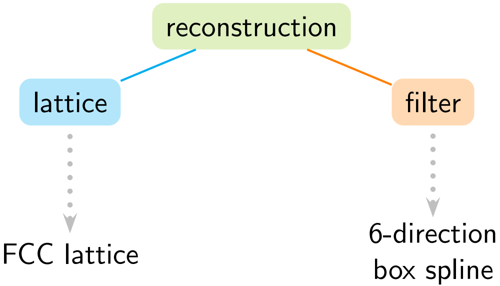

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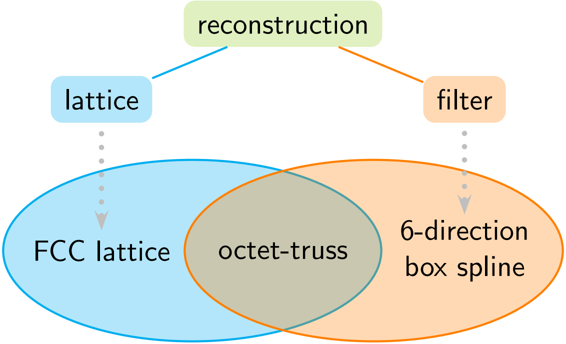

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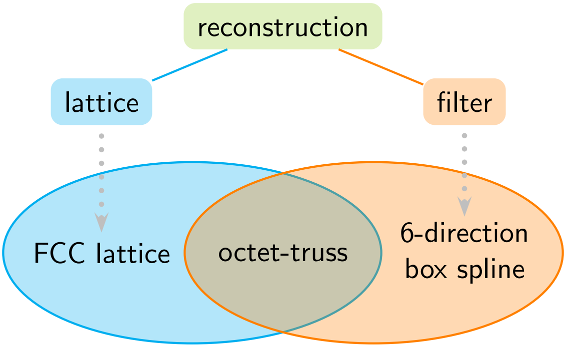

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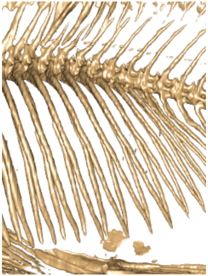
- Integrated with POV-Ray ray-tracer and **source codes** are freely available at `http://www.cise.ufl.edu/research/SurfLab/08vis`.

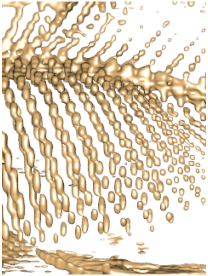
Example

original



100%

standard method

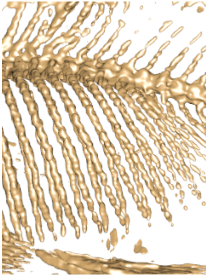


6%

Cartesian lattice

tri-quadratic B-spline

our approach



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FCC lattice

6-direction box spline

Example

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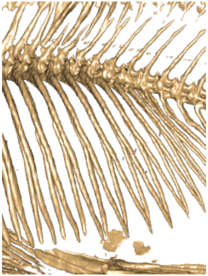

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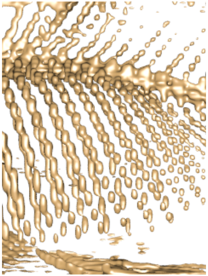

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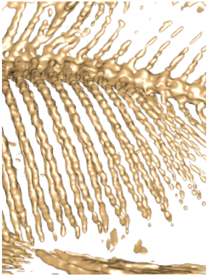


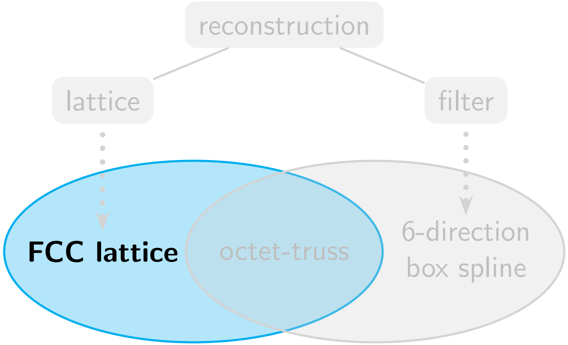
6%

FCC lattice

6-direction box spline

For (random) evaluation, our approach is **20%** faster!

Sampling Lattice: FCC Lattice



FCC Lattice: Definition

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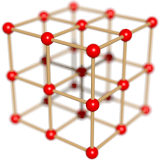

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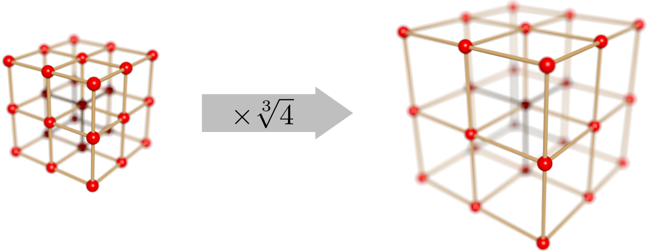

Cubic (Cartesian) lattice

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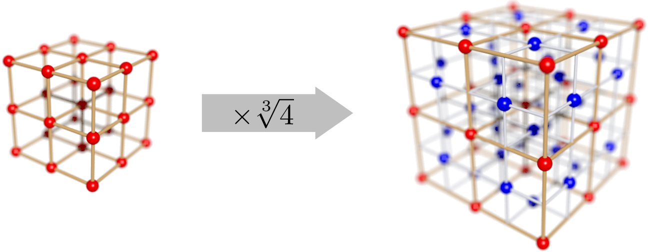

Cubic (Cartesian) lattice $+$ additional facet points

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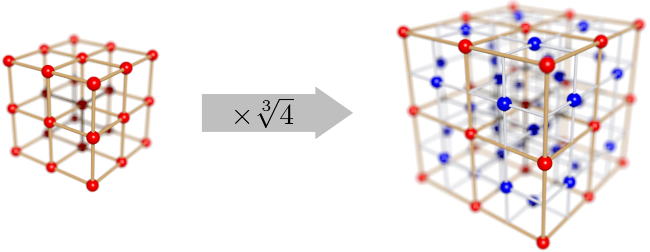

Cubic (Cartesian) lattice $+$ additional facet points
$\rightarrow$ "Face-Centered Cubic" lattice.

FCC Lattice: Voronoi Cell

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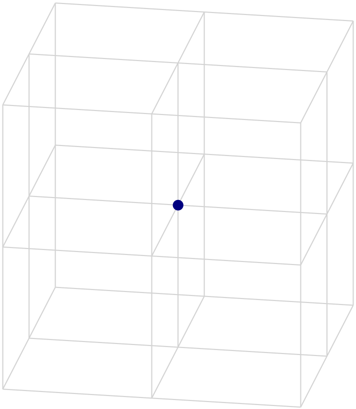

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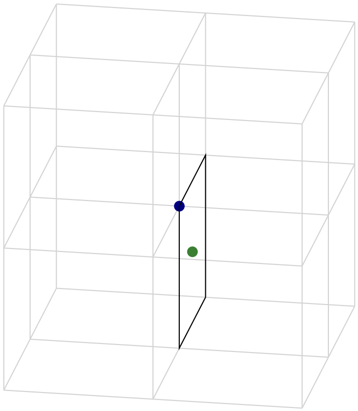

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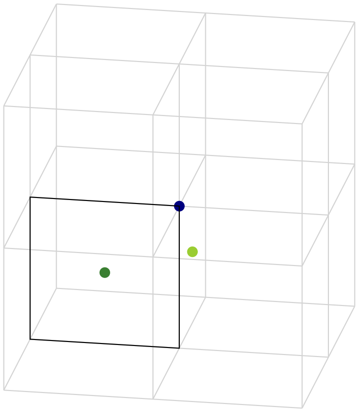

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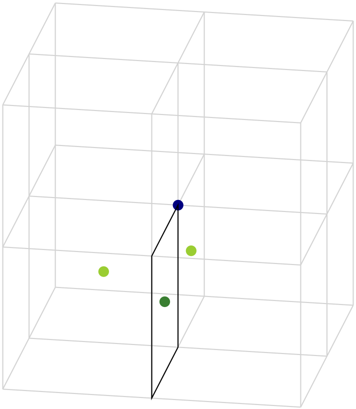

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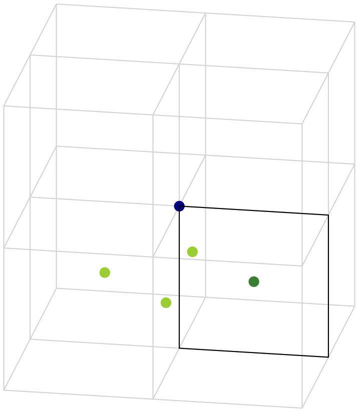

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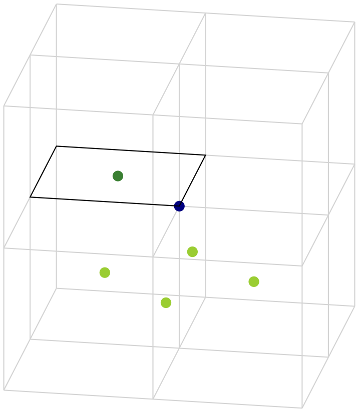

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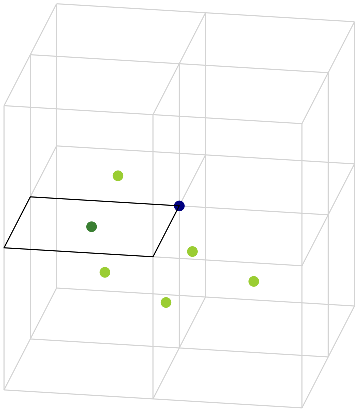

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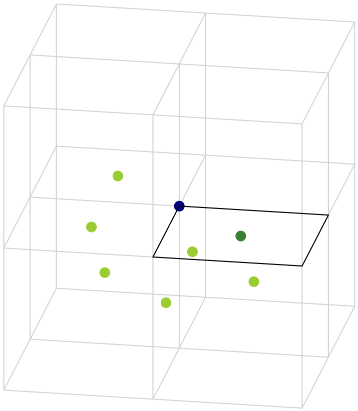

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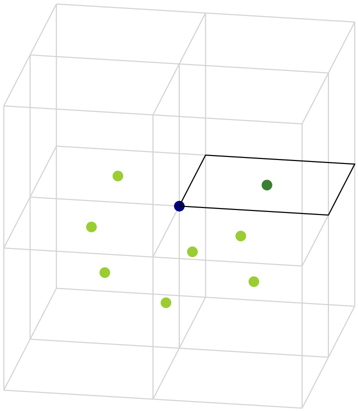

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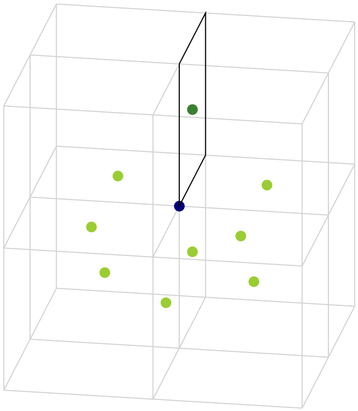

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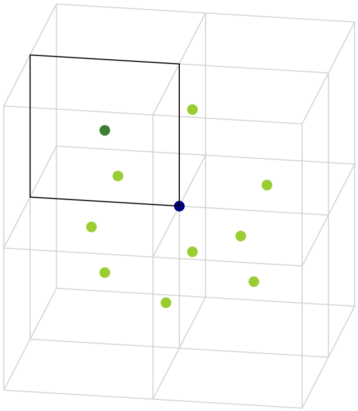

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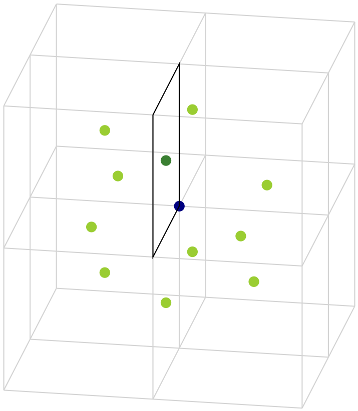

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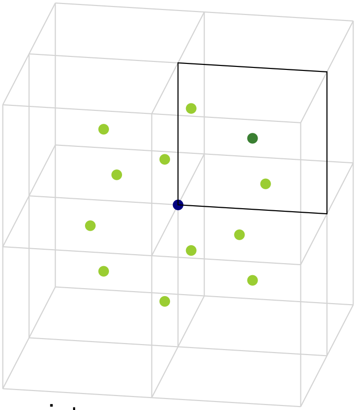

12 nearest neighbor points

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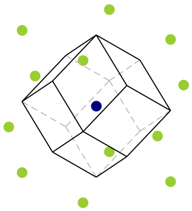

12 nearest neighbor points
$\rightarrow$ Voronoi cell $=$ **Rhombic Dodecahedron**.

FCC Lattice: Applications

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- Sampling efficiency: Cartesian $<$ FCC $<$ BCC. (Petersen & Middleton '62)

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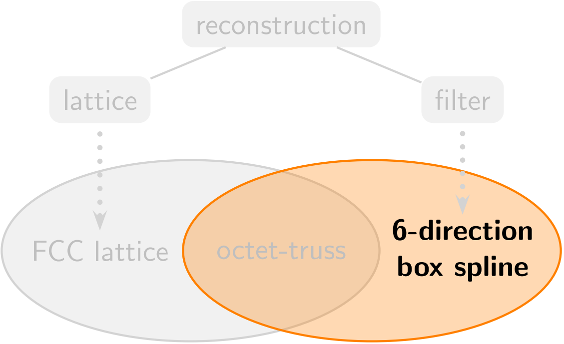
Efficient sampling: minimizes number of samples necessary to reconstruct an isotropic band-limited signal.

- Multiresolution data structure (Inoue et al. 2008), Global illumination (Qiu et al. 2007),

Reconstruction Filter: 6-Direction Box Spline



Box-Splines: A Bivariate Example

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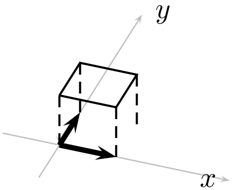
Direction matrix $\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$



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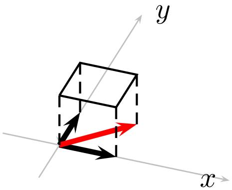

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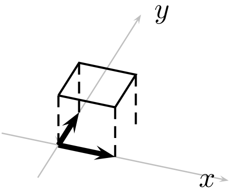

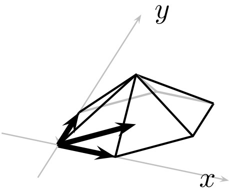

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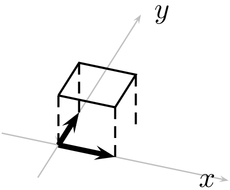

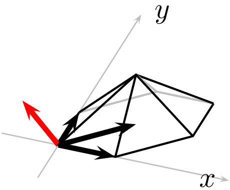

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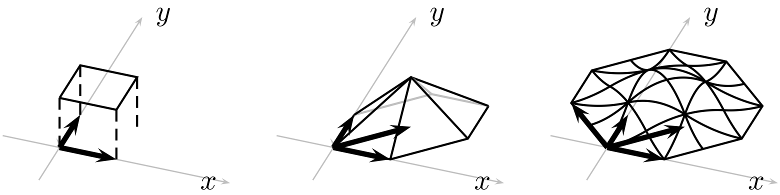

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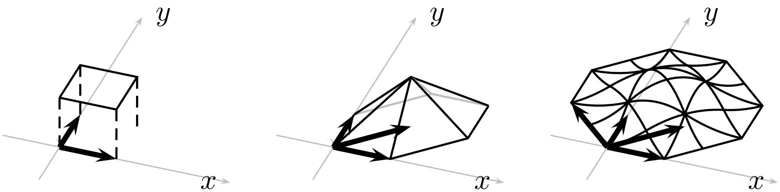

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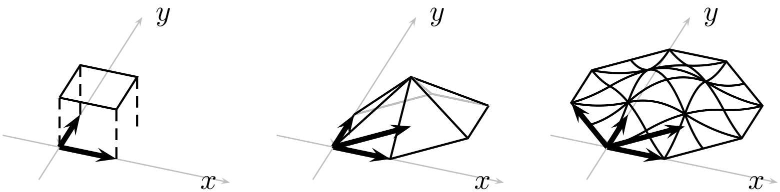

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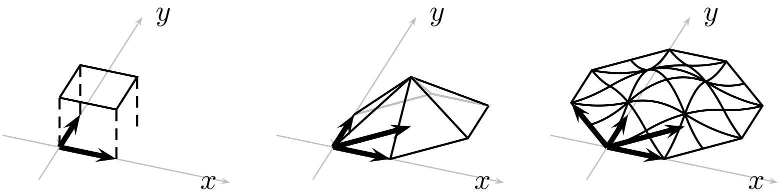

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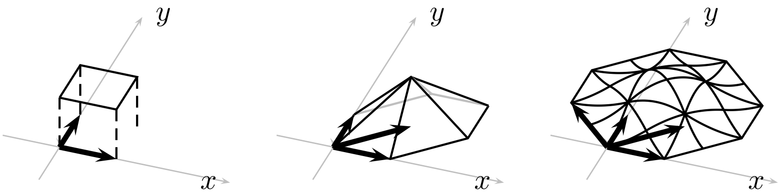

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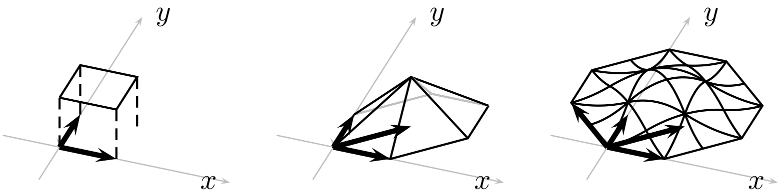

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- "Box Splines" (Carl de Boor et al., 1993).

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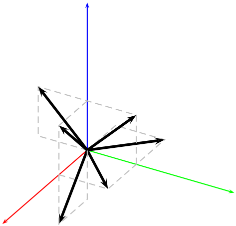

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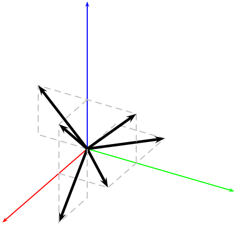
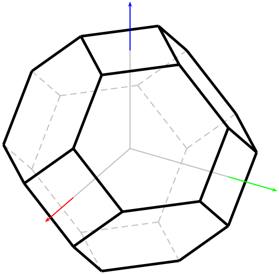
6-Direction Box Spline





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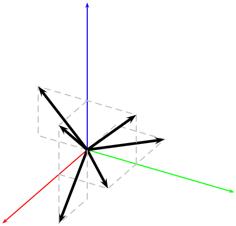
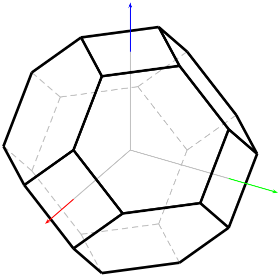
- Support $=$ **Truncated Octahedron**.

6-Direction Box Spline





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- Total degree cubic and $\mathbf{C^1}$ **continuous**.

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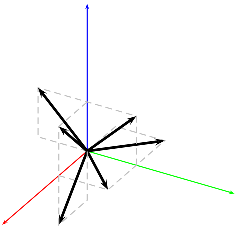

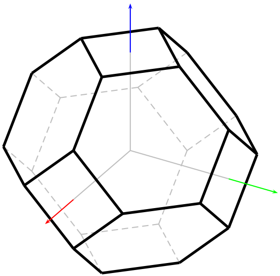

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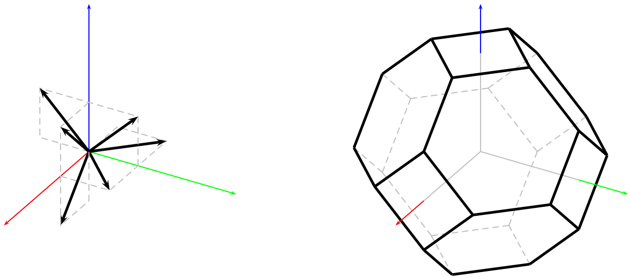

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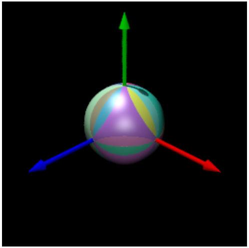
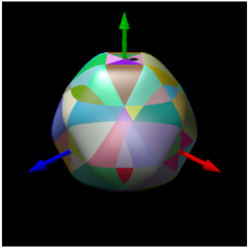
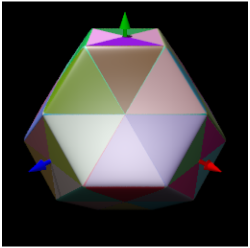
- **Exact rational coefficients** are pre-computed.

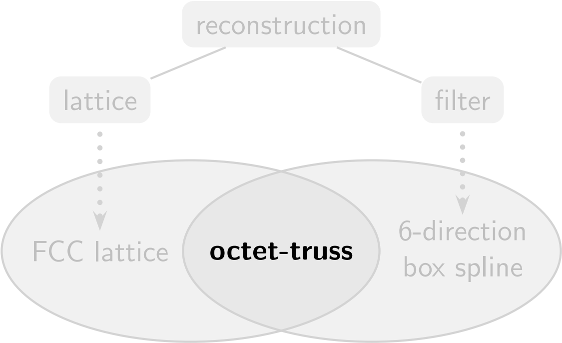
6-Direction Box Spline (cont'd)







Polynomial Structure: Octet-Truss



Octet-Truss Structure

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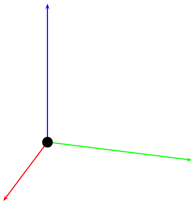

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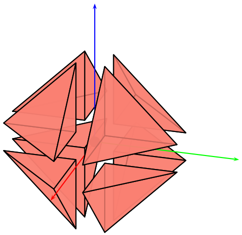

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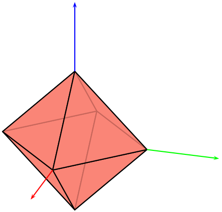

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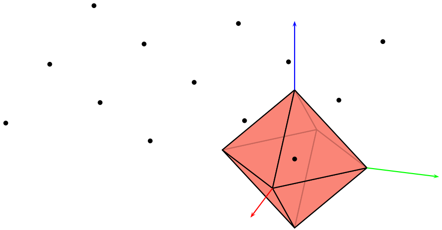

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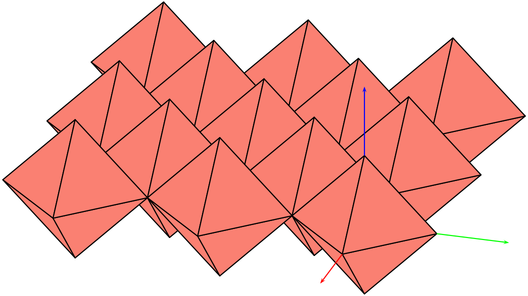

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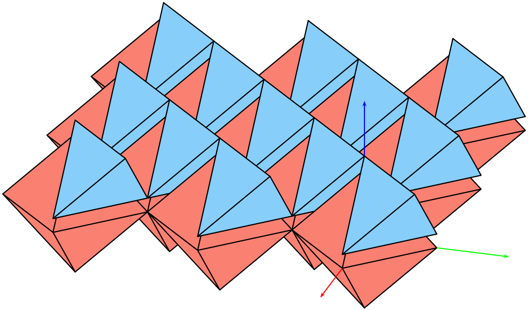
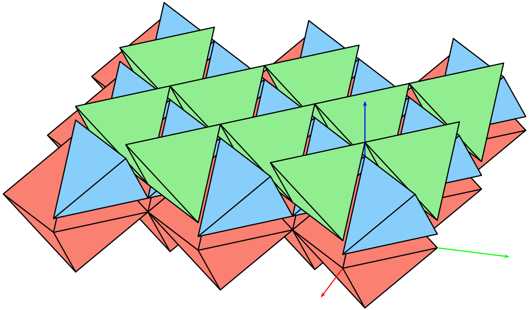
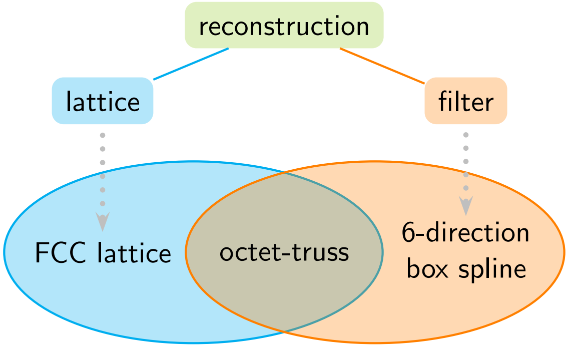


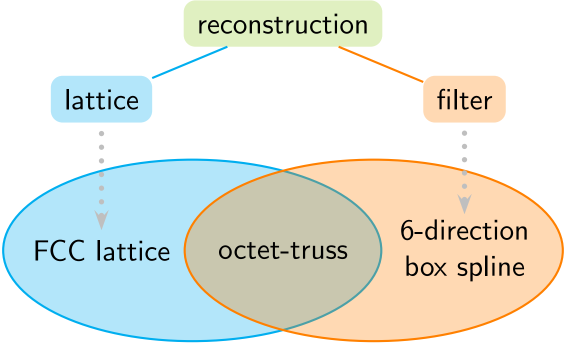
Octet-Truss Structure



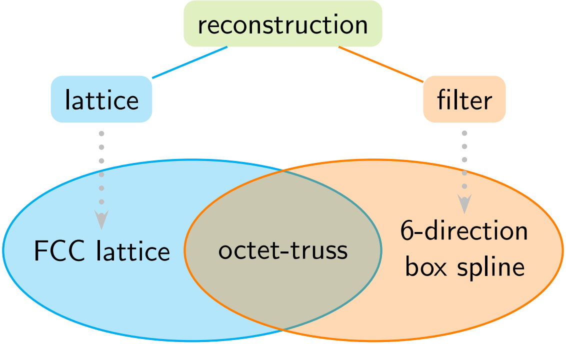
6-Direction Box Spline on the FCC Lattice



6-Direction Box Spline on the FCC Lattice



6-Direction Box Spline on the FCC Lattice



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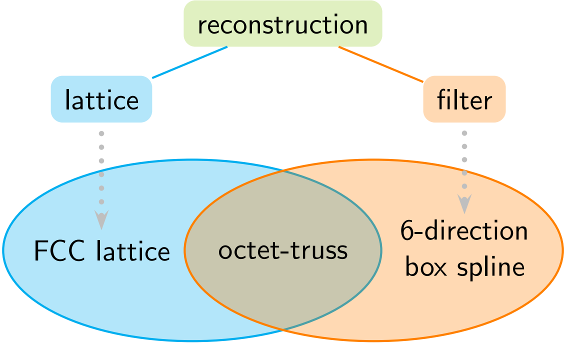

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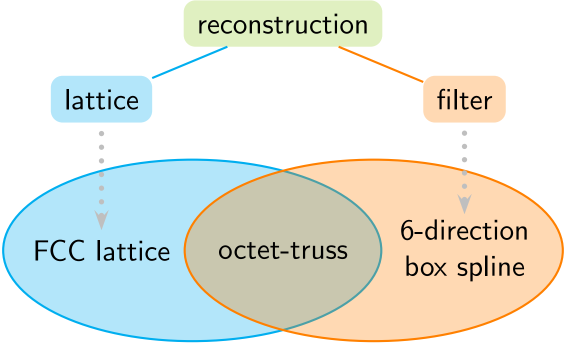

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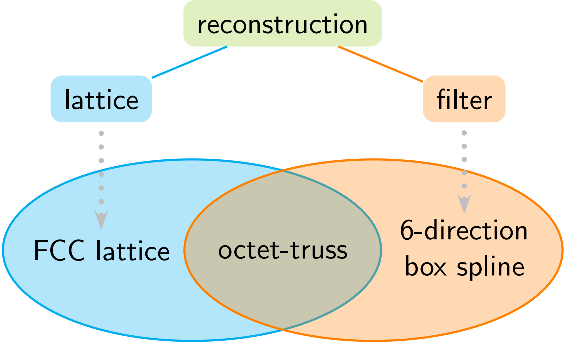
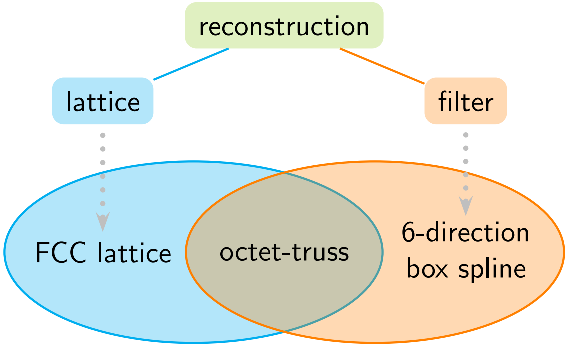
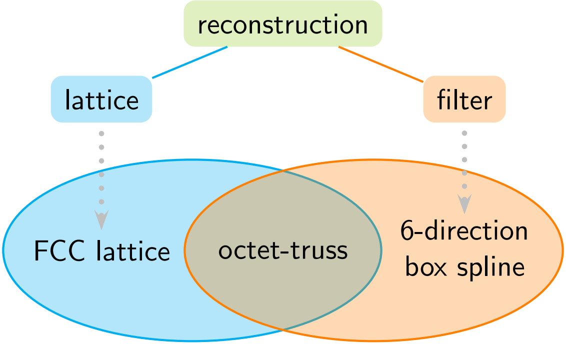


6-Direction Box Spline on the FCC Lattice



- Polynomial structure $\longrightarrow$ octet-truss structure.

6-Direction Box Spline on the FCC Lattice



- Polynomial structure → octet-truss structure.
- Shifts are linearly independent → *basis* functions.

Comparison: Math

Comparison: Math

	Standard	Our approach
lattice	Cartesian	FCC
filter	tri-quadratic B-spline	6-direction box spline

Comparison: Math

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polynomial structure	**cubes**	octet-truss

Comparison: Math

	Standard	Our approach
lattice	Cartesian	FCC
filter	tri-quadratic B-spline	6-direction box spline
polynomial structure	**cubes**	octet-truss
approximation order	**3**	**3**

Comparison: Math

	Standard	Our approach
lattice	Cartesian	FCC
filter	tri-quadratic B-spline	6-direction box spline
polynomial structure	**cubes**	octet-truss
approximation order	**3**	**3**
total degree	6	**3**

Comparison: Math

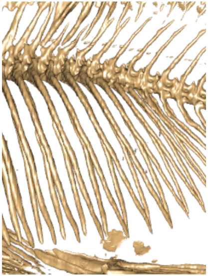
	Standard	Our approach
lattice	Cartesian	FCC
filter	tri-quadratic B-spline	6-direction box spline
polynomial structure	**cubes**	octet-truss
approximation order	**3**	**3**
total degree	6	**3**
stencil size	27	**16**

Comparison: Math

	Standard	Our approach
lattice	Cartesian	FCC
filter	tri-quadratic B-spline	6-direction box spline
polynomial structure	**cubes**	octet-truss
approximation order	**3**	**3**
total degree	6	**3**
stencil size	27	**16**
sampling efficiency	poor	**good**

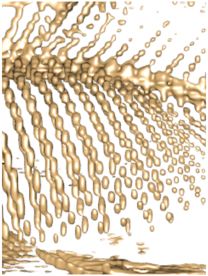
Comparison: Reconstuction (Carp dataset)

original



100%

standard method

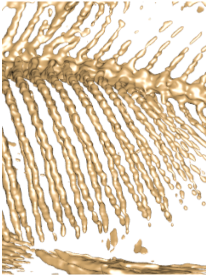


6%

Cartesian lattice

tri-quadratic B-spline

our approach



6%

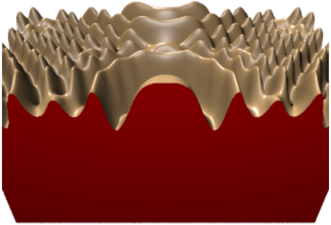
FCC lattice

6-direction box spline

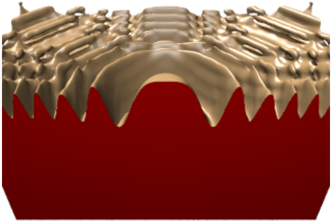
Comparison: Reconstruction (Marschner-Lobb function)

Comparison: Reconstruction (Marschner-Lobb function)

density 0.07^{-3}



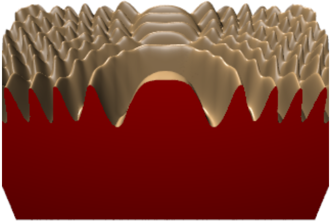
Standard



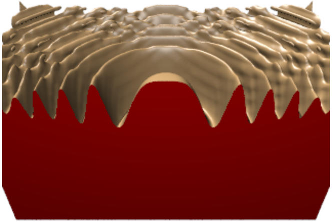
Our approach

Comparison: Reconstruction (Marschner-Lobb function)

density 0.06^{-3}



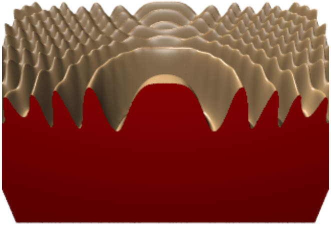
Standard



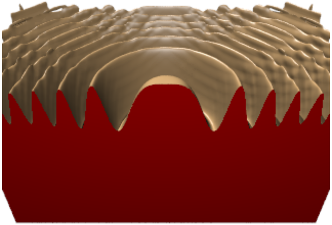
Our approach

Comparison: Reconstruction (Marschner-Lobb function)

density 0.05^{-3}



Standard



Our approach

Comparison: Computation Time

Dataset	Standard	Our approach	Ratio
Marschner-Lobb	135	98	**72%**
Carp	515	358	**69%**

- Rendering time (in seconds) to generate ray-casted images.

Try yourself!

For more information, please visit

`http://www.cise.ufl.edu/research/SurfLab/08vis`

Thank you!

Selected References

Carl de Boor, Klaus Höllig, and Sherman Riemenschneider, *Box splines*, Springer-Verlag New York, Inc., New York, NY, USA, 1993.

Minho Kim and Jörg Peters, *Fast and stable evaluation of box-splines via the BB-form*, Numerical Algorithms (2008), in print.

Daniel P. Petersen and David Middleton, *Sampling and reconstruction of wave-number-limited functions in N-dimensional euclidean spaces*, Information and Control **5** (1962), no. 4, 279–323.